(7 pages) **Reg. No. :** ....

Code No. : 20383 E **Sub. Code : CAMA 21**

B.Sc.(CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second/Fourth Semester

Mathematics — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The unit normal vector to the surface $x^3 - xyz^3 + z^3 = 1$ at $(1, 1, 1)$ is ————

(a) $\dfrac{2\vec{i} - \vec{j} + 2\vec{k}}{3}$ (b) $2\vec{i} - \vec{j} + 2\vec{k}$

(c) $\dfrac{\vec{i} - 2\vec{j} + 2\vec{k}}{3}$ (d) $\vec{i} + 2\vec{j} + 3\vec{k}$

2. If $\vec{F} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ solenoidal then the value of 'a' is ————

(a) 2 (b) -2

(c) 1 (d) 0

3. The value of $\int_0^1\int_0^1 (x^2+y^2)\,dxdy$ is ————

(a) $\dfrac{2}{3}$ (b) 2

(c) $\dfrac{1}{2}$ (d) 2

4. The value of $\int_0^a\int_0^a\int_0^a dzdydx$ is

(a) a^3 (b) a^2

(c) a (d) 1

5. $\int_0^{\pi/2} \left(3\sin x\vec{i} + 2\cos x\vec{j}\right) dx =$ ————

(a) $3\vec{i} + 2\vec{j}$ (b) $3\vec{i} - 2\vec{j}$

(c) $-3\vec{i} + 2\vec{j}$ (d) $-3\vec{i} - 2\vec{j}$

6. If S is the sphere $x^2+y^2+z^2=1$, the value of $\iint_S \vec{r}.\hat{n}ds$ is

(a) $\frac{4\pi}{3}$ (b) 3π

(c) 4π (d) 2π

7. If C is the circle $x=\cos\theta$, $y=\sin\theta$ then $\int_C (xdy-ydx)=$ ————————

(a) π (b) $\frac{\pi}{2}$

(c) 2π (d) $\frac{\pi}{4}$

8. $\iint_C \vec{F}.d\vec{r}=\iint_S (\nabla\times\vec{F})\hat{n}\,ds$ is

(a) Fundamental theorem

(b) Gauss-divergence theorem

(c) Green's theorem

(d) Stoke's theorem

9. If $f(x)=\begin{cases}1, & 0<x<\pi\\ 0, & \pi<x<2\pi\end{cases}$ then the value of the Fourier coefficinet a_n is

(a) 0 (b) –2

(c) –3 (d) –4

10. If $f(x)=|x|$ in $(-\pi,\pi)$ then the fourier coefficient a_0 is

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) $\frac{\pi^2}{2}$

PART B — ($5\times5=25$ marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Obtain the directional derivative of $\phi=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$

Or

(b) Prove that curl grad $\phi=\nabla\times\nabla\phi=0$.

[P.T.O.]

12. (a) Evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ using change of order of integration.

Or

(b) Evaluate $\int_0^{\log a} \int_0^x \int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$

13. (a) Evaluate $\int_C \vec{f}.d\vec{r}$ where $\vec{f} = (x^2+y^2)\vec{i} + (x^2-y^2)\vec{j}$ and C is the curve $y = x^2$ joining (0, 0) and (1,1)

Or

(b) Evaluate $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ for a vector S $x^2+y^2+z^2 = a^2$ in the upper hemisphere and $z \geq 0$, $\iint_S (\nabla \times \vec{F})\hat{n}\,ds$.

14. (a) Use Green's theorem to evaluate $\int_C (x^2y\,dx + y^3\,dy)$. where C is the closed path formed by $y = x$ and $y = x^3$ from (0,0) to (1,1).

Or

(b) Evaluate $\int_C (e^x dx + 2y\,dy - dz)$, by using stoke's theorem where C is the curve $x^2+y^2 = 4$, $z = 2$.

15. (a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$

Or

(b) Find a sine series for $f(x) = c$ in the range 0 to π

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that $\nabla \times (\nabla \times \vec{f}) = \nabla(\nabla.\vec{f}) - \nabla^2 f$

Or

(b) If $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. If $\vec{F} = \nabla\phi$ then find the value of ϕ.

17. (a) Find the area of the region D bounded by the parabolas $y = x^2$ and $x = y^2$.

Or

(b) Evaluate $\iint_D x^2y^2\,dx\,dy$, where D is the circular disc $x^2+y^2 \leq 1$.

18. (a) Find the work done by the force $\vec{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

Or

(b) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$

19. (a) Verify Gauss theorem for $\vec{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken over the cuboid $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$

Or

(b) Verify stoke's theorem for $\vec{F} = (x^2 - y^2)\bar{i} + 2xy\bar{j}$ int he rectangular region $x = 0$, $x = a$, $y = 0$, $y = b$.

20. (a) Express $f(x) = \frac{1}{2}(\pi - x)$ as a fourier series with period 2π, to be valid in the interval $(0, 2\pi)$

Or

(b) Find a cosine series in the range 0 to π for

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$$